ED-322

M.Sc. 1st Semester
Examination, March-April 2021

COMPUTER SCIENCE

Paper - I

Mathematical Foundation of
Computer Science

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Prove that :

(*i*) $\sim(p \vee q) \Leftrightarrow (\sim p) \wedge (\sim q)$

(*ii*) $\sim(p \wedge q) \Leftrightarrow (\sim p) \vee (\sim q)$

(*b*) Explain the following terms and also give example to explain them :

(*i*) Quantifier

(*ii*) Negation of a Quantifier

(*c*) Let T be the set of all triangles in a plane and $R = \{(a, b) \mid \text{area of } \Delta a = \text{area of } \Delta b\}$; that is $a\,R\,b$ if and only if area of Δa = area of Δb. Prove that R is an equivalence relation.

Unit-II

2. (*a*) A lattice L is distributive if and only if $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a)$ $\forall\ a, b, c, \in L$.

(*b*) Let $(L, \vee, \wedge)$ be an algebraic system, where $\vee$ and $\wedge$ are binary operations satisfying the absorption law. Show that $\vee$ and $\wedge$ also satisfy the idempotent law.

(*c*) If $(B, +, \cdot, ')$ is a Boolean algebra, then prove that the following statements are equivalent :

(*i*) $a \cdot b' = 0$

(*ii*) $a + b = b$

(*iii*) $a' + b = 1$

(*iv*) $a \cdot b = a$

Unit-III

3. (*a*) Prove that the inverse of the product of two elements of a group is the product of the inverse taken in the reverse order i.e. $(ab)^{-1} = b^{-1} a^{-1}$ $\forall\ a, b \in G$.

(*b*) Define Grammar. Find the phase-structure grammar that generate the set :

$$L = \{a^n b^{2n}, n \geq 1\}$$

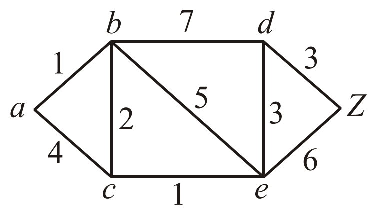
(*c*) Show that the order of a subgroup of a finite group divides the order of the group.

Unit-IV

4. (*a*) Prove that in any graph, the number of vertices of odd degree is always even.

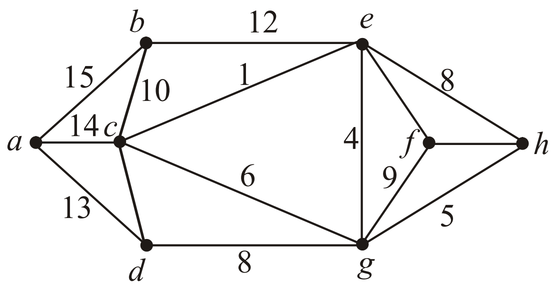
(*b*) Prove that if the intersection of two path in a graph is a disconnected graph. Show that the union of the two path has at least one circuit.

(*c*) Write an algorithm for shortest path in weighted graph and use it to find shortest path from *a* to *z* in the graph shown in figure where number associated with the edges are the weights.



Unit-V

5. (*a*) Find the minimum spanning tree for the graph :



(*b*) Prove that A tree with n vertices has $(n - 1)$ edges.

(*c*) Express the following algebraic expression in binary tree :

(*i*) $(x - y) + ((y + z) + w)$

(*ii*) $(((a \times b) + c) - d) \times (e + f)$