Roll No..................... **Total No. of Questions : 05**

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Code No. : B-271(A)

Annual Examination - 2017

B.Sc.-III

MATHEMATICS

Paper-I

ANALYSIS

Max.Marks : 50

Time : 3 Hrs. **Min.Marks : 17**

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¹ãq ß Zà½u§ý ¢§ýà¢ y ¥§ý ZàÎÂà Ñv §ýLâk¥ ñ ysã ZàÎÂàà §ý ¡§ý ytàÂà Ñ ñ

Note : Attempt one question from each unit. All questions carry equal marks.

Unit-I

ZàÎÂà-1. (¡) âÂàÈÂàâvâhm Äývâà §ýL ÄýáÊuÊ ó½àã ZààÃm §ýLâk¥ ß

$f(x)=x^2$ kÑà mnà

¡m¥w ó½àã $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+----$ §ýà tàÂà ZààÃm §ýLâk¥ ñ

Find the Fourier series of the function :

$f(x)=x^2$, where $-\pi \le x \le \pi$ and $f(x+2\pi)=f(x)$.

Hence find the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+----$.

P.T.O.

(r) Ðw°²¦ qÀà¦ §ýL óâ½àuà¦ §ý ¡âsyÊ½à §ý âv¥ ¡àr¦v qÊãÕà½à §ýà §ýnÂà âvâh¥ ñ ¢y§ýL yÑàumà y¦ ây÷ §ýLâk¥ â§ý ó¦½àã

¡âsyàÊã Ñ¦ ñ

Write Abel's test for convergence of arbitrary series. Using this test show that the series

is convergent.

(y) tàÂàà â§ý

mr qâÊsàxà y¦

$f_x(0,0)$, $f_y(0,0)$, $f_{xx}(0,0)$, $f_{yy}(0,0)$ mnà $f_{xy}(0,0)$ §ýà tàÂà Öààm §ýLâk¥ ñ

Suppose that $f(x,y)=\begin{cases} \dfrac{xy\left(x^2-y^2\right)}{x^2+y^2} & \text{when } (x,y)\neq(0,0) \\ 0 & \text{when } (x,y)=(0,0) \end{cases}$

then from definition, evaluate

$f_x(0,0)$, $f_y(0,0)$, $f_{xx}(0,0)$, $f_{yy}(0,0)$ and $f_{xy}(0,0)$

Define continuity and uniform continuity of a function on a metric space (X,d). Show by an example, that every continuous function is not uniformly continuous.

(r) tàÂàvà¦ ÀÊã§ý ytâ¯ Ñ¦ mnà ÀÊã§ý

âÂàÊÂààÂàyàÊ qâÊsààxm Ñ¦ ß $d_1(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$, $\forall\, x,y\in X$

mr âÀhà¢¥ â§ý ÀÊã§ý d_1 ¡àÊ mÌu-ÀÊã§ý Ñ¦ ñ

Let be a metric space and metric is defined as follows : $d_1(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$, $\forall\, x,y\in X$. Then show that metrices d_1 and d are the equivalent metrices.

(X,d) $d: X\times X\to R$ $1+\dfrac{1}{4.3}+\dfrac{1}{4^2.5}+\dfrac{1}{4^3.7}+\dfrac{1}{4^4.9}+\ldots$

$f(x,y)=\begin{cases} \dfrac{xy\left(x^2-y^2\right)}{x^2+y^2} & \text{kr } (x,y)\neq(0,0) \\ 0 & \text{kr } (x,y)=(0,0) \end{cases}$

(y) Zà½u§ý yÑm ÀÊã§ý ytâ¯ q½à¦ Ñàmã Ñ¦, ây÷ §ýLâk¥ ñ

"Every compact metric space is complete". Prove.

---x---

(y) tàÂàvà d wàÐmâw§ý y©uà yt°ju qÊ yàtàÂu ÀÊã§ý Ñ mnà ¡àÊ $B=(3, 5]$ mr âÂàÈÂààâ§ým §ýà tàÂà Öààm §ýLâk¥ ß

i) $\delta(A)$ ii) $\delta(B)$ iii) $d\left(\frac{5}{2}, A\right)$

iv) v)

kÑà íuày, yt°juà §ý tÁu ÀÊã Ñ ñ

Let be a usual metric on a set of real numbers R, and and $B=(3, 5]$ then evaluate the following :

i) $\delta(A)$ ii) $\delta(B)$ iii) $d\left(\frac{5}{2}, A\right)$

iv) v)

where is diameter and D is distance between sets.

Unit-V

ZàÎÂà-5. (¡) yàm¼u ¡àÊ ¥§ý ytàÂà yàm¼u ÄývÂà §ýL qâÊsàxà, ÀÊã§ý ytâ˜ §ý âv¥ Àãâk¥ ñ ¥§ý £ÀàÑÊ½à y âÀhà¢¥ â§ý Zà¼u§ý ymmi Äývâà; ¥§ý ytàÂà ymm âàÑã Ñàmà ñ

Unit-II

ZàÎÂà-2. (¡) ÄývÂà §ý âv¥ ÀÎàà¢u â§ý mnà $\int_0^a x^2\,dx = a^3/3$.

For the function $f(x)=x^2, \forall\, x \in [0, a]$, $a>0$ show that .

(r) â»ÊÛýv qÊãÕà½à §ýà §ýnÂà âvâh¥ ñ ytà§ývÂà $\int_a^\infty \frac{\sin x}{\sqrt{x}}\,dx$ kÑà $a>0$ §ý ¡âsyÊ½à §ýà qÊãÕà½à §ýLâk¥ ñ

[illegible]

State the Dirichlet's test. Test the convergence of the integral , where $a>0$.

(y) ÄývÂàã ytà§ývÂà §ý £quàªà y âý÷ §ýLâk¥ â§ý ß

Using Frullani's integral prove that :

Unit-III

zâÎÀà-3. (¡) yâtó ÄývÂà §ý âwÎvâx§ý ÑàÂà §ý âv¥ ¡àwÎu§ý "§ýàÎàã-Êãtàâà" ¡ààÎà§ý ¡w§ývÂà ytã§ýÊ½à §ýà âvâh¥ ñ ¢y ây÷ §ýLâk¥ ñ

State the necessary condition of "Cauchy-Riemann" partial differential equation for a function $f(z)=u(x,y)+iv(x,y)$ to be analytic. Prove this condition.

(r) ¥§ý âõÊhãu ÛýqàÂmÊ½à Öààm §ýLâk¥ ârÀ¡à $0,1,\infty$ §ýà §ýtÎàß qÊ Zââmâ j âØàm §ýÊmà Ñ ñ

Find the bilinear transformation which maps the points to the point respectively.

(y) âõÊhãu ÛýqàÂmÊ½à §ý ÔÐnÊ ârÀ ¡àÊ yªàm Zàyàtààu Ûýq Öààm §ýLâk¥ ñ

Find the fixed point and corresponding normal form to the bilinear transformation .

Unit-IV

zâÎÀà-4. (¡) ÀÊã§ý ytâ˜ §ýL qáÊsàxà âvâh¥ ñ ¥§ý ÀÊã§ý ytâ˜ t ây÷ §ýLâk¥ â§ý .

Define metric space. In a metric space (X,d)

Prove that :

$|d(x,z)-d(y,z)| \le d(x,y), \forall x,y \in (X,d)$

(r) ytâ˜ l^{∞} ysã qáÊr÷ wàÐmâw§ý ¡àà§ýtà §ýà yt°ju Ñ ñ tàÂàvà , $y=\{y_n\}_{n=1}^{\infty}$ ¢y§ý Àà Ðw°² ârÀ Ñ âkyt ÀÊã§ý âÂàÊÂààÂàyÊ qáÊsààxm Ñ-

$d(x,y)=\sup\{|x_n-y_n| : n\in N\}, \forall x,y\in l^{\infty}$

mà âÀhà¢¥ â§ý ¥§ý ÀÊã§ý ytâ˜ Ñ ñ

Space l^{∞} is a set of all bounded real number's sequences. Suppose , $y=\{y_n\}_{n=1}^{\infty}$ are two arbitary sequences points, in which the metric is defined as follows:

$d(x,y)=\sup\{|x_n-y_n| : n\in N\}, \forall x,y\in l^{\infty}$. Then show that is a metric space.