Roll No...................... **Total No. of Questions : 03**

Total No. of Printed Pages : 05

Code No. : B-272(A)

Annual Examination - 2017

B.Sc.-III

MATHEMATICS

Paper-II

ABSTRACT ALGEBRA

Max.Marks : 50

Time : 3 Hrs. **Min.Marks : 17**

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¹¦ãq ß Zà¾u§ý ¢§ýà¢ y¦ ¥§ý ZàÎÂà Ñv §ýLâk¥ ñ ysã ZàÎÂàà¦ §ý ¡§ý ytàÂà Ñ ñ

Note : Attempt one question from each unit. All questions carry equal marks.

$Aut(\delta)$

Unit-I

ZàÎÂà-1. (¡) **ây÷ §ýLâk¥ â§ý â§ýyã ytÑ δ §ý ysã ¡àmáÊ§ý Ðw§ýàáÊmà ¡à¦ §ýà ytº ju §ýà ¥§ý Zàyàtàu £qytÑ Ñàmà Ñ ¡àÊ uÑ §ý âwsàªà ytÑ y¦ mÌuà§ýàÊã Ñàmà Ñ, kÑà §ýà §ýÂô Ñ ñ**

Prove that the set of all inner automorphism of a group is a normal subgroup of and is isomorphic to the quotinet group of where is the centre of .

P.T.O.

(r) tàÂàvà ¥§ý qâÊâtm ytÑ Ñ ñ t §ý yuªtã ¡wuwà §ýL y©uà t §ý Zàyàtàu§ý §ýà yjj§ý Ñàmà Ñ ¡nàm

Let δ be a finite group, The numbe of elements conjugate to a in is the index of the normalizer of

a in ie.

(y) âõmãu âyvà Zàtu âvâh¥ ¥w ây÷ §ýLâk¥ ñ

State and prove second Sylow's theorem.

Unit-II

ZàÎÂà-2. (¡) ây÷ §ýLâk¥ â§ý â§ýyã wvu R §ýL Àà ªà½àkàwâvuà §ýà ywâÂà™ §ýL ¥§ý ªà½àkàwvã Ñàmà Ñ ñ

Prove that the intersection of two idelas of any ring R is an ideal of R.

(r) ây÷ §ýLâk¥ â§ý ¥§ý ¥yà m½yt§ýL §ýt-âwâÂàtu wvu Ñ âky§ý ªà½àkàwvã uà Ðwu Ñà mà ¥§ý Õàøà Ñ ñ

If is such a commutative ring with unity whose ideal is (0) on itself then prove that is a field.

(y) tàÂàvà ¥§ý -tà»Üv ¡mÕàqã ¥§ý -tà»Üv §ýL ¥§ý ytà§ýâáÊmà Ñ, mà §ýà ¥§ý -£qtà»Üv Ñàmà Ñ ñ

$C_a = \frac{o(\delta)}{o(N(a))}$

Unit-V

ZàÎÂà-5. (¡) Îwàk ¡yàt§ýà âvâh¥ ¥w ây÷ §ýLâk¥ ñ

State and prove Schwarz's Inequality.

(r) ây÷ §ýLâk¥ â§ý ¥§ý ¡àÂmÊ ªà½àÂà ytâ˜ t ÎàÂuÙàÊ yàÀÎàà §ýà §ýà¢ vàÒÈr§ý yt°ju Êâh§ýmß Ðwmøà Ñàmà Ñ ñ

Prove that any orthogonal set of non-zero vectors in an inner produced space V is linearly independent.

(y) ªààt-ÒÎt¹ §ý vàÒÈr§ý Zà§ýt §ýà £quàªà §ýÊ§ý ¡àoàÊ y ¥§ý Zàyàtàu vàÒÈr§ý ¡àoàÊ ZààÂm §ýLâk¥ kÑà $\alpha=(1,0,0), \beta=(1,1,0)\, \gamma=(1,1,1)$.

Using Gram-Schmidt orthogonalization process obtain an orthonormal basis from the basis $B=\{\alpha, \beta, \gamma\}$ of $V_3(R)$ where $\beta=(1,1,0)\, \gamma=(1,1,1)$

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Let $f : M \to N$ be an homorphism of an module M into an module N, then the kert is an submodule of M.

Unit-III

ZàÎÂà-3. (¡) ây÷ §ýLâk¥ â§ý â§ýyä yâÀÎà ytâ˜| §ý ¥§ý ¡âÊQý £qyt°ju §ý âv¥ §ýà ¥§ý £qyât˜| ÑàÂà| §ý âv¥ ¡àwÎu§ý ¥w| quàÃm Zàâmr|o Ñ| ß

i)

ii)

Prove that the necessary and sufficient conditions for a non-empty subset w of a vector space to be a subspace of V are

i)

ii)

$W \in V, \beta \in F \Rightarrow a\alpha + \beta \in w$

$\dim \frac{V}{w} = \dim V - \dim w$

(r) ây÷ §ýLâk¥ â§ý yâÀÎà mnà

$\gamma = (0, -3, 2)$ $V_3(R)$ §ýà ¡àoàÊ rÂààm| Ñ| ñ

Prove that the vectors $\alpha = (1, 0, -1)$, $\beta = (1, 2, 1)$ and $\gamma = (0, -3, 2)$ $V_3(R)$ form a basis of $V_3(R)$

(y) uâÀ ¥§ý qáÊâtm âwtäu yâÀÎà ytâ˜| §ýà ¥§ý £qytâ˜|

Ñ| mà|

(3) **Code No. : B-272(A)**

Let $f : M \to N$ be an homorphism of an module M into an module N, then the kert is an submodule of M.

Unit-III

ZàÎÂà-3. (¡) ây÷ §ýLâk¥ â§ý â§ýyä yâÀÎà ytâ˜| §ý ¥§ý ¡âÊQý £qyt°ju §ý âv¥ §ýà ¥§ý £qyât˜| ÑàÂà| §ý âv¥ ¡àwÎu§ý ¥w| quàÃm Zàâmr|o Ñ| ß

i)

ii)

Prove that the necessary and sufficient conditions for a non-empty subset w of a vector space to be a subspace of V are

i)

ii)

(r) ây÷ §ýLâk¥ â§ý yâÀÎà mnà

$\gamma = (0, -3, 2)$ $V_3(R)$ §ýà ¡àoàÊ rÂààm| Ñ| ñ

Prove that the vectors $\alpha = (1, 0, -1)$, $\beta = (1, 2, 1)$ and $\gamma = (0, -3, 2)$ $V_3(R)$ form a basis of $V_3(R)$

(y) uâÀ ¥§ý qáÊâtm âwtäu yâÀÎà ytâ˜| §ýà ¥§ý £qytâ˜|

Ñ| mà|

It w is a subspace of a finite dimensional vector space then

$V_3(F)\to V_2(F)$ $\dim \frac{V}{W} = \dim V - \dim W$ $y^2 + 3z^2 - 4yz + 6zx$

Unit-IV

ZàÎÂà-4. (¡) uâÀ $f:V_3(F)\to V_2(F)$ âÂàÈÂà Zà§ýàÊ y¦ qáÊsààxm Ñ¦ $f(x,y,z)=(y,z)$ mà¦ âÀhà¢¥ â§ý f ¥§ý Êâh§ý Úqà¦mÊ½à Ñ¦ ñ

If is defined as $f(x,y,z)=(y,z)$ then show that *f*, is linear transformation.

(r) ÀÎàà¢u¦ â§ý tâ¹¦¨y âw§ý½àJu Ñ¦ ß

Show that the matrix A is diagonalizable :

$$A=\begin{bmatrix}3 & 2 & 4\\ 2 & 0 & 2\\ 4 & 2 & 3\end{bmatrix}$$

(y) âÂàÈÂà âõ¡àmã yt¡àm §ýà¦ âwâÑm Úq t¦ ÍuQý §ýLâk¥ mnà £y§ýL kààm, y¦j§ýà§ý ¥w¦ â¦j0ÂÑ§ýà Öààm §ýLâk¥ ñ

Redue the following quadratic form in into cononical form and find its rank, index and signature.

(4) **Code No. : B-272(A)**

It w is a subspace of a finite dimensional vector space then

Unit-IV

ZàÎÂà-4. (¡) uâÀ $f:V_3(F)\to V_2(F)$ âÂàÈÂà Zà§ýàÊ y¦ qáÊsààxm Ñ¦ $f(x,y,z)=(y,z)$ mà¦ âÀhà¢¥ â§ý f ¥§ý Êâh§ý Úqà¦mÊ½à Ñ¦ ñ

If is defined as $f(x,y,z)=(y,z)$ then show that *f*, is linear transformation.

(r) ÀÎàà¢u¦ â§ý tâ¹¦¨y âw§ý½àJu Ñ¦ ß

Show that the matrix A is diagonalizable :

$$A=\begin{bmatrix}3 & 2 & 4\\ 2 & 0 & 2\\ 4 & 2 & 3\end{bmatrix}$$

(y) âÂàÈÂà âõ¡àmã yt¡àm §ýà¦ âwâÑm Úq t¦ ÍuQý §ýLâk¥ mnà £y§ýL kààm, y¦j§ýà§ý ¥w¦ â¦j0ÂÑ§ýà Öààm §ýLâk¥ ñ

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